

A Study of Even Development and Sustainable Development

MASAAKI ABE

.INTRODUCTION

There are many definitions of sustainable development, and they often incompatible with each other. At the risk of oversimplification, we can distinguish two broad ideological groups in environmentalism. One group pursues expansion of consumption for economic growth, and the other group aims to limit consumption for conservation of environment. The efforts to the opposite direction from each other are equal to doing nothing. That is why we are not able to progress to sustainable development.

We need to find out the goal of sustainable development. To discuss the goal as a whole is beyond me. We advance an argument focusing on economic growth. Some people regard sustainable development as continuative economic growth. The other people argue steady-state economy is necessary for sustainable development. There are some people, moreover, who think reductions in the scale of economy are required for sustainable development. Which position is right?

The purpose of this paper is to find out the goal of sustainable development and possibility of even-development between the North-South by analyzing the simple dynamic model of economic growth in the very long run within the extended framework of Krugman (1981). The model portrays a two-region world in which the industrial sectors of regions grow through the accumulation of capital. There are two crucial assumptions. One is external economies in the industrial sector. Therefore a small 'head start' for one region will cumulate over time, with exports of manufactures from the leading region crowding out the industrial sector of the lagging region. Second is essentialness of renewable resource for manufactured production. In this model, renewable resource plays crucial role. It is the source of manufactured production and it restrict eternal growth.

.THE MODEL

We consider a world consisting of two regions, North and South. To sharpen the analysis, we assume that these regions are identical in the sense that technological and behavioral relationships are the same. We also assume that the regions have equal labor forces L and that these labor forces do not grow over time. Thus we have

$$(1) \quad L_N = L_S = \bar{L}.$$

Each region is able to produce two goods, a manufactured good M and an

agricultural product A, and to trade at zero transportation cost. There will thus be a single world price of manufactured goods in terms of agricultural products, P_M . Agricultural products are produced by labor alone; we will choose units so that one unit of labor produces one unit of agricultural goods.

The growth sector, however, is manufacturing. Manufacturing requires capital K, labor L and renewable resource R. We assume that, from the point of view of an individual firm, the unit capital, labor and renewable resource requirements are fixed. In the aggregate, however, unit capital, labor and renewable resource requirements are not constant; instead, in each region they are decreasing functions of the region's aggregate capital stock. Letting c , v and r are the unit capital, labor and renewable resource requirements, respectively, we have

$$(2) \quad \begin{aligned} c_N &= c(K_N), & c_S &= c(K_S), \\ v_N &= v(K_N), & v_S &= v(K_S), \\ r_N &= r(K_N), & r_S &= r(K_S), \end{aligned}$$

Where c' , v' , $r' < 0$. However, we assume that the absolute value of the elasticity of unit input requirements with respect to output is less than one, so that total input requirements rise as manufacturing output rises. In order to simplify an analysis, we also assume neutral technical growth($c: v: r = \text{constant}$).

At the initial point, we assume that labor force and renewable resource for manufacturing are abundant. Given the relationships (2), then, together with full employment of factors, we can determine the pattern of output. In each region the output of manufactured goods depends on the capital stock:

$$(3) \quad M_N = \frac{K_N}{c(K_N)}, \quad M_S = \frac{K_S}{c(K_S)}.$$

Given the output of manufactured goods, then, total requirements of labor and renewable resource with respect to output are given by the following forms: $v_N M_N$, $v_S M_S$ and $r_N M_N$, $r_S M_S$.

Output of agricultural goods can then be determined from the agricultural sector's role as a residual claimant on labor:

$$(4) \quad A_N = \bar{L} - v_N M_N, \quad A_S = \bar{L} - v_S M_S.$$

Next, we define the nature of the renewable resources. With the initial point when both regions just began accumulation of capital, We assume that Initial endowment of the renewable resource R^0 is abundant and upper limit. The amount of the renewable resource of both regions does not exceed R_{\max} . To simplify the analysis, the endowment of both regions is equal at the initial point:

$$(5) \quad R_N^0 = R_S^0 = R_{\max}.$$

The stock of renewable resource will be decreased by input to industrial production and will increase by reproduction of these selves. To keep the model simple, we assume that the renewable resource has a certain fixed growth rate a ($0 < a < 1$) and it is used only for industrial production. When the amount of resource is R , the increment of next term becomes aR . Hence, if the input

becomes larger than aR , the amount of renewable resource of next term will decrease. Thus, we can describe the change of each region's stock of renewable resources as

$$(6) \quad \dot{R}_N = aR_N - r_N M_N, \quad \dot{R}_S = aR_S - r_S M_S.$$

Note that the conditions of $R_N, R_S \leq R_{\max}$ exist in (6).

There are two upper limits $K_{\max L}, K_{\max R}$. $K_{\max L}$ is the amount of capital, which comes when the region is completely specialized in manufacturing and no more labor can be drawn out of agriculture. $K_{\max R}$ is the amount of capital, which comes when the input of renewable resources reaches the upper limit R_{\max} and no more resources can be gathered. We can define $K_{\max L}, K_{\max R}$ as follows:

$$(7) \quad \frac{v(K_{\max L}) \cdot K_{\max L}}{c(K_{\max L})} = \bar{L}, \quad \frac{r(K_{\max R}) \cdot K_{\max R}}{c(K_{\max R})} = R_{\max}$$

Consider next the distribution of income. There are two cases: the case in which at least some labor is used in agricultural production, and the case of complete specialization in manufacturing. If some labor is used in agriculture, this ties down the wage rate, which is 1 in terms of agricultural goods, $1/P_M$ in terms of manufactures. We can then determine the rental per unit of capital as a residual. For simplicity, let us assume (though it is not essential) that capital goods are produced by labor alone, i.e., we include them as part of 'agricultural' output. Using the price of manufactured good P_M and the cost of gathering renewable resource C_R , then the rental per unit of capital, measured in agricultural (or wage) units, is also the profit rate, and we have

$$(8) \quad \rho_N = \frac{P_M - v_N - r_N C_R^N}{c_N}, \quad \rho_S = \frac{P_M - v_S - r_S C_R^S}{c_S}$$

Where ρ_N, ρ_S are profit rates North and South. We assume that the input cost of renewable resources in each region C_R depends on the amount of renewable resources in each region, and they are decreasing functions of the amount of renewable resources in each region.

$$(9) \quad \begin{aligned} C_R^N &= C(R_N) \quad , \quad C'(R_N) < 0 \\ C_R^S &= C(R_S) \quad , \quad C'(R_S) < 0 \end{aligned}$$

Since c and v are functions of the capital stocks and C_R is a function of the amount of renewable resources, we can also write (8) as a pair of reduced form equations

$$(10) \quad \rho_N = \rho(P_M, K_N, R_N) \quad , \quad \rho_S = \rho(P_M, K_S, R_S)$$

where $\partial \rho / \partial P_M > 0$, $\partial \rho / \partial K > 0$ and $\partial \rho / \partial R > 0$, the second expression is caused by increasing returns to scale in manufacturing.

When a region is completely specialized in manufacturing, (10) no longer holds. Instead the rate of profit is determined in Kaldorian fashion by the requirement that savings equal zero, if there is no foreign investment, or by the rate of profit on foreign investment if there is such investment. In the latter case the wage rate is residually determined.

To close the model we need to specify the demand side. We will make two

strong assumptions for the sake of easy algebra; the conclusions of the model could be derived under weaker but less convenient assumptions. First, saving behavior is classical: all profits and only profits are saved. Second, a fixed proportion μ of wages will be spent on manufactures, $1-\mu$ on agricultural goods.

The savings assumption means that, if there is no international investment, the rate of growth of the capital stock in each region will just equal the rate of profit.

$$(11) \quad \frac{\dot{K}_N}{K_N} = \rho_N \quad , \quad \frac{\dot{K}_S}{K_S} = \rho_S$$

The relative price of manufactured goods will be determined by world demand and supply. Since a fraction μ of wages is spent on manufactures, provided that both countries produce some agricultural goods, and we have

$$(12) \quad P_M [M_N + M_S] = \mu [L_N + L_S]$$

which can be rewritten as

$$(13) \quad P_M = \frac{2\mu\bar{L}}{[K_N/c(K_N) + K_S/c(K_S)]} \equiv P_M(K_N, K_S)$$

This gives us a relationship between the two capital stocks and P_M ; it is apparent that P_M is decreasing in both capital stocks. Note also that K_N and K_S enter symmetrically, so that where $K_N=K_S$, $P_M/K_N = P_M/K_S$.

Finally, we can combine (10), (11) and (13) to express the rate of change each region's capital stock as a function of the levels of both capital stocks and renewable resources:

$$(14) \quad \frac{\dot{K}_N}{K_N} = \rho(P_M, K_N, R_N) \equiv g_N(K_N, K_S, R_N)$$

$$\frac{\dot{K}_S}{K_S} = \rho(P_M, K_S, R_S) \equiv g_S(K_S, K_N, R_S)$$

We can confirm the characters of these functions as follows:

$$(15) \quad \frac{\partial g_N}{\partial K_S} = \frac{\partial \rho}{\partial P_M} \cdot \frac{\partial P_M}{\partial K_S} < 0$$

$$\frac{\partial g_S}{\partial K_N} = \frac{\partial \rho}{\partial P_M} \cdot \frac{\partial P_M}{\partial K_N} < 0$$

$$\frac{\partial g_N}{\partial K_N} = \frac{\partial \rho}{\partial P_M} \cdot \frac{\partial P_M}{\partial K_N} + \frac{\partial \rho}{\partial K_N}$$

$$\frac{\partial g_S}{\partial K_S} = \frac{\partial \rho}{\partial P_M} \cdot \frac{\partial P_M}{\partial K_S} + \frac{\partial \rho}{\partial K_S}$$

$$\frac{\partial g_N}{\partial R_N} = \frac{\partial \rho}{\partial \mathcal{C}_R^N} \cdot \frac{\partial \mathcal{C}_R^N}{\partial R_N} > 0$$

$$\frac{\partial g_S}{\partial R_S} = \frac{\partial \rho}{\partial \mathcal{C}_R^S} \cdot \frac{\partial \mathcal{C}_R^S}{\partial R_S} > 0$$

The effect of an increase in the other region's capital stock must be to turn the terms of trade against manufactures and thus reduce profits; so $g_2 < 0$. The

effect of an increase in the amount of renewable resource must also increase profits of own region through a decrease of the gathering cost; so $g_3 > 0$. The effect of an increase in the domestic capital stock is, however, ambiguous, since there are two effects: a worsening of the terms of trade and a reduction in unit input requirements. If the scale effects which is positive to profits stronger than the price effects which is negative to profit, the equilibrium in this dynamic growth model is consistently unstable in local (A formal proof is given in the appendix). Therefore, in the case that the price elasticity of manufactured good is low, we have a conclusion that uneven development is a necessary outcome in this model. So we proceed with the analysis by taking assumption that external economies are relatively weak: $g_1 < 0$.

$$(16) \quad \frac{\partial g_N}{\partial K_N} = \frac{\partial \rho}{\partial P_M} \cdot \frac{\partial P_M}{\partial K_N} + \frac{\partial \rho}{\partial K_N} < 0$$

$$\frac{\partial g_S}{\partial K_S} = \frac{\partial \rho}{\partial P_M} \cdot \frac{\partial P_M}{\partial K_S} + \frac{\partial \rho}{\partial K_S} < 0$$

We have now set out a complete dynamic model in which the evolution of the two regions' industrial sectors can be followed from any initial position. The next step is to trace out and interpret the path of the world economy over time.

. THE DYNAMICS

The equilibrium of this model is given by the point that $g_N=0$, $g_S=0$. The lines $g_N=0$, $g_S=0$ indicate combinations of K_N and K_S for which profits in North and South respectively are zero. Given the assumptions in section 2, each line has a downward sloping.

$$(17) \quad \frac{dK_N}{dK_S} = -\frac{\partial g_N / \partial K_S}{\partial g_N / \partial K_N} < 0 \quad (\text{on the line of } g_N=0)$$

$$\frac{dK_N}{dK_S} = -\frac{\partial g_S / \partial K_S}{\partial g_S / \partial K_N} < 0 \quad (\text{on the line of } g_S=0)$$

And it is clear that the line $g_N=0$ is steeper than the line $g_S=0$. When the point which represent the capital of each region is inner from the each line $g=0$ at the $K_S K_N$ -plane, then $g > 0$. Reversely, when the point is outer from the each line $g=0$, then $g < 0$.

The line $g=0$ is affected by the amount of renewable resource. We know that the effect of an increase in the amount of renewable resource must also increase profits of own region through a decrease of the gathering cost. Therefore, the line $g=0$ will move outside (inside) if the amount of renewable resources increase (decrease).

Next, we find out the level of capital K , which remains amount of renewable resources in next team at every given level of renewable resources. From the definition of the renewable resources, we have

$$(18) \quad K_N = a \frac{c_N}{r_N} R_N \equiv \tilde{K}_N(R_N) \quad , \quad K_S = a \frac{c_S}{r_S} R_S \equiv \tilde{K}_S(R_S)$$

We call this line “ $h=0$ ”. Now, we can check changes of the amount of renewable resources on the $K_S K_N$ -plane. It is clear that the amount of the renewable resources will increase (decrease) when the point, which represents the capital of each region, is inner (outer) from the line $h=0$ at the $K_S K_N$ -plane.

We consider the relative locations of the two kind of lines $g=0$, $h=0$. It is natural to think that the amount of renewable resources is affected by manufactured production. Actually, we are having experience of depletion all over the world. Therefore, we assume that line $h=0$ is located inner from the line $g=0$.

Now, we have a figure (FIGURE 1) that illustrates the essential point of this dynamic growth model, and that shows us the level of capital accumulation and amount of renewable resources in each region.

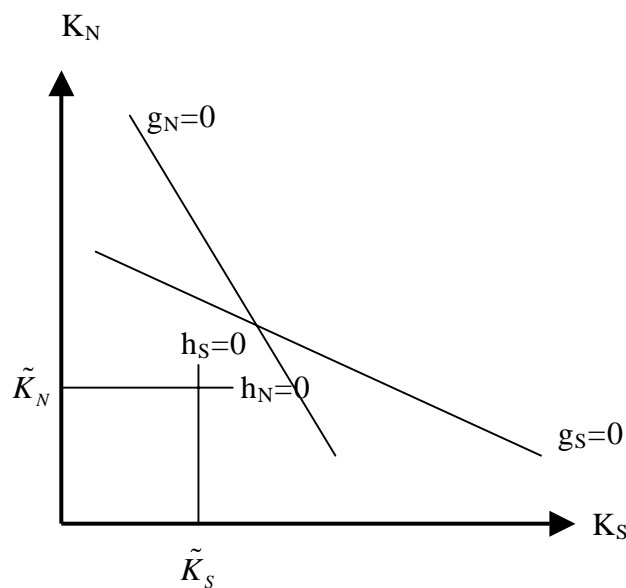


FIGURE 1

The line of profit zero ($g=0$) and the line of resource change zero ($h=0$)

Note that the lines in this figure continuously be moved. For example, the lines are not moved during the capital in north region K_N is less than \tilde{K}_N . For the amount of renewable resources is on an upper limit at the initial point and it stays unchanged. But once K_N exceed \tilde{K}_N , which means inputs for manufactured goods exceed the regeneration of renewable resources, the lines $g_N=0$, $h_N=0$ will move inside for the decrease of renewable resources. This movement represents that the progress of capital accumulation causes increase of renewable resources input for manufactured goods, and decrease of renewable resources will happen after $K_N = \tilde{K}_N$. After that the profit rate from manufactures good and the regeneration ability of renewable resources will decrease resulting from a rise of gathering cost of renewable resources.

The point of figure 1 is that the direction of capital accumulation is directly controlled by the relative location of capital stocks with the line of

profit zero ($g=0$). Furthermore, the line of resource change zero ($h=0$) affects indirectly the direction of capital accumulation through the movement of the line of profit zero ($g=0$).

Level of K_N	Movement of line $h_N=0$	Movement of line $g_N=0$	Change of resources
$K_N > \tilde{K}_N$	downward	down ward	decrease
$K_N < \tilde{K}_N$	up ward	up ward	increase

Level of K_S	Movement of line $h_S=0$	Movement of line $g_S=0$	Change of resources
$K_S > \tilde{K}_S$	left ward	left ward	decrease
$K_S < \tilde{K}_S$	right ward	right ward	increase

TABLE 1

Movement of the figure 1 relative with the location of capital stocks

. Dynamics of sustainable development

In this chapter, we analyze the dynamic process of capital accumulation in two regions since the beginning of a situation, which has abundant renewable resources and slight capital accumulation in both regions.

First, we consider a case of no renewable resource trade. There are two representative dynamic processes, which depend on manufactured good price elasticity of renewable resources. If the elasticity is sufficiently high, the change of profit rate is larger than the change of renewable resource. That is the line $g=0$ moves faster than the line $h=0$. Conversely, if the manufactured good price is inelastic, the line $h=0$ moves faster than the line $g=0$. Figure 2 and figure 3 show the dynamic process of these two cases.

In the case of elastic (FIGURE 2), we began to analyze from the point 0, at that point the North has a small head start to the South. The profit rate is positive in each region. So, both regions progress the capital accumulation. Note that the small head start cumulates for the present for scale economy. After that, the North arrives at the line of resource change zero ($h_N=0$). And the renewable resources in the North began to decrease (the line $h_N=0$ be moved to downward). It causes that decrease of the profit rate in the North (the line $g_N=0$ be moved to downward) and the capital accumulation in the North gradually gets slowdown. Finally, the point of capital accumulation in the North arrives at the line of profit zero ($g_N=0$), and the capital turn to decrease from this point.

The other side, the South comes later to the North in same process. Now, the point in figure 2, which represents the level of capital accumulation in each region, takes a process from northeast direction to southwest direction. It means that both regions accumulate their capitals at first, and after that, their capitals

turn to be decreased by declines of profit rate for decrease of renewable resources in each region.

Next, it happens that a change of direction again. As the decrease of capital, the point arrives at the line $h=0$, and the renewable resources turns to increase. After that, profit rate is increased and it is started to accumulate capital.

As mentioned above, the following process is repeated.

1. Decrease of renewable resources by overreach accumulation of capital.
2. Decrease of capital.
3. Increase of renewable resources.
4. Accumulation of capital.

And, the point of capital accumulation spirally convergence to the central equilibrium point. This equilibrium is stable.

In the case of inelastic (FIGURE 3), the decrease of capital does not enough speedy to recover the renewable resource. After all, Capital in each region decreases until it becomes zero.

We consider next a tradable case of renewable resource. We assume trade cost is zero. In this case, the input costs of renewable resources in both regions become same. We assume that the input cost of renewable resources depends on the total amount of renewable resources in each region. And we also assume the cost increase for scarcity of renewable resources.

$$(19) \quad C_R^N = C_R^S = C_R \equiv C_R(R) \\ C_R'(R) < 0 \quad (\text{Where } R = R_N + R_S)$$

Next, we consider the line $h=0$. it needs the condition of input equal reproduction of renewable resources for zero change of the amount of renewable resources.

$$(20) \quad r(K_N) \frac{K_N}{c(K_N)} + r(K_S) \frac{K_S}{c(K_S)} = a(R)$$

From the assumption neutral technical growth ($c:v:r=\text{constant}$), we have a expression for the line $h=0$ common to both regions.

$$(21) \quad K_N = -K_S + a \frac{c}{r} R$$

The dynamic process of this case is presented in FIGURE 4. In this case, the decrease of renewable resources in one region causes decrease the both regions profit. The result is an ever increasing divergence between the regions, which end only when the renewable resource are exhausted.

. Conclusion

The two regions model shows that necessary condition for even development is high elasticity of manufactured good price. And the case of no resource trade has higher possibility for even development compare with the case of trading in resource.

The result of the case of even development leads to the conclusion that infinite accumulation of capital is impossible on condition that finite growth of resources. Therefore, the sustainable growth is to attain steady state economy in which fixed amount of resources is thrown into production of a manufactured good and fixed amount of manufactured good are produced in every period.

Appendix

Local stability of equilibrium

1. The case of no renewable resources trade

We have the system of dynamic model with in this paper as follows.

$$(A1) \quad \begin{aligned} \frac{\dot{K}_N}{K_N} &\equiv g_N(K_N, K_S, R_N) & , & & \frac{\dot{K}_S}{K_S} &\equiv g_S(K_S, K_N, R_S) \\ \frac{\dot{R}_N}{R_N} &\equiv h(K_N, R_N) & , & & \frac{\dot{R}_S}{R_S} &\equiv h(K_S, R_S) \end{aligned}$$

The Jacobian matrix of this system is as follows.

$$(A2) \quad J = \begin{vmatrix} \frac{\partial g_N}{\partial K_N} & \frac{\partial g_N}{\partial K_S} & \frac{\partial g_N}{\partial R_N} & 0 \\ \frac{\partial g_S}{\partial K_N} & \frac{\partial g_S}{\partial K_S} & 0 & \frac{\partial g_S}{\partial R_S} \\ \frac{\partial h_N}{\partial K_N} & 0 & \frac{\partial h_N}{\partial R_N} & 0 \\ 0 & \frac{\partial h_S}{\partial K_S} & 0 & \frac{\partial h_S}{\partial R_S} \end{vmatrix}$$

The following formula is realized at the equilibrium by the symmetry of two regions and neutral technical growth.

$$(A3) \quad \frac{\partial g_N}{\partial K_N} = \frac{\partial g_S}{\partial K_S} , \quad \frac{\partial g_N}{\partial K_S} = \frac{\partial g_S}{\partial K_N} , \quad \frac{\partial h_N}{\partial K_N} = \frac{\partial h_S}{\partial K_S} , \quad \frac{\partial g_N}{\partial R_N} = \frac{\partial g_S}{\partial R_S}$$

The following formula is obtained by calculating the Jacobian using (A3).

$$(A4) \quad J = \left(\frac{\partial g_N}{\partial K_N} \cdot \frac{\partial h_N}{\partial R_N} - \frac{\partial h_N}{\partial K_N} \cdot \frac{\partial g_N}{\partial R_N} \right)^2 > 0$$

The divergence of the Jacobian matrix (A2) is as follows.

$$(A5) \quad D = 2 \left(\frac{\partial g_N}{\partial K_N} + a \right) \quad (\quad h_N/ \quad R_N = a)$$

Therefore, Jacobian always takes positive value. Then, the value of the divergence must be negative for local stable equilibrium. Now we have following condition as stability of equilibrium.

$$(A6) \quad \frac{\partial g_N}{\partial K_N} < -a$$

There were two effects to profit rate: external effect and price effect. When the price effect is relatively strong, the following condition is satisfied: $g_N/K_N < 0$. Therefore, we can confirm that the more manufactured good price elasticity of resources is high, the more stability of equilibrium increases.

2. The tradable case of renewable resources with no cost

We have the system of dynamic model with in this paper as follows.

$$(A7) \quad \begin{aligned} \frac{\dot{K}_N}{K_N} &\equiv g_N(K_N, K_S, R) \quad , \quad \frac{\dot{K}_S}{K_S} \equiv g_S(K_S, K_N, R) \\ \frac{\dot{R}}{R} &\equiv h(K_N, K_S, R) \quad \quad \quad (\text{Where } R = R_N + R_S) \end{aligned}$$

The stability condition are obtained in the same way of case 1 as follows:

$$(A8) \quad -2 \cdot \frac{\partial h}{\partial K_N} \cdot \frac{\partial g_N}{\partial R_N} > a \left(-\frac{\partial g_N}{\partial K_S} - \frac{\partial g_N}{\partial K_N} \right) \quad (J > 0)$$

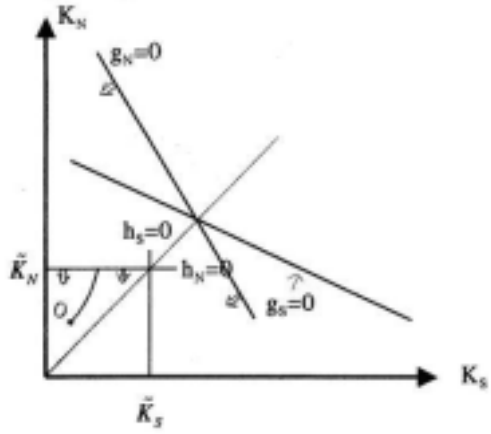
$$2 \cdot \frac{\partial g_N}{\partial K_N} + a < 0 \quad (D < 0)$$

Therefore the condition of negative divergence is higher manufactured good price elasticity of renewable resources. And there is the condition of positive Jacobian for stable equilibrium. If $J < 0$ equilibrium is a saddle point.

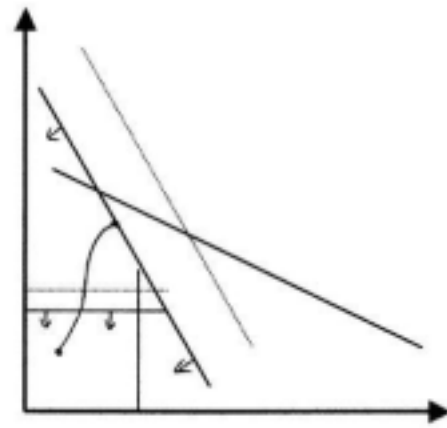
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Krugman, Paul, "Trade, Accumulation, and Uneven Development" Journal of Development Economics, vol.8, no.2, Apr. 1981, pp.149-161.

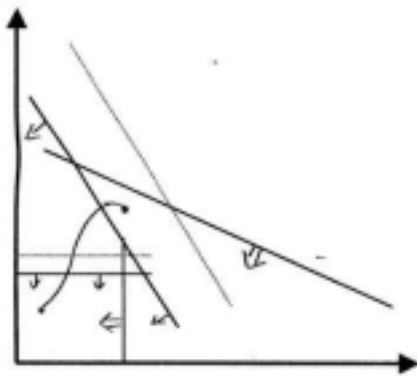
2.1 Beginning of resources decrease in North



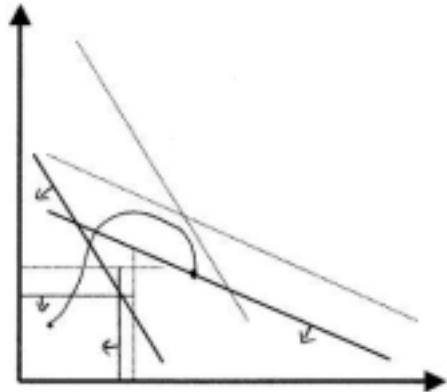
2.2 Profit in North turns to minus



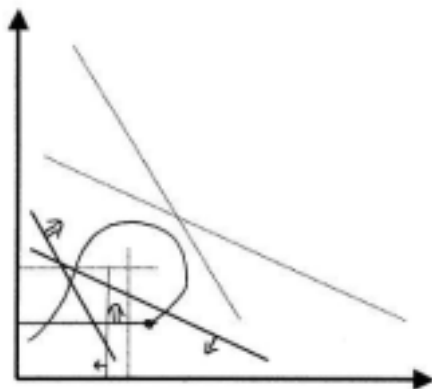
2.3 Beginning of resources decrease in South



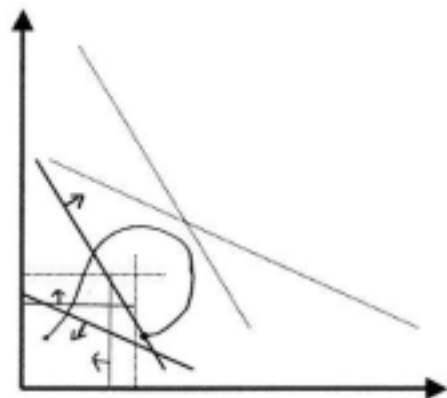
2.4 Profit in South turns to minus



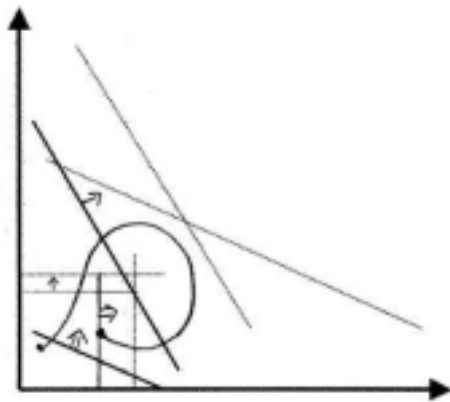
2.5 Beginning of resources increase in North



2.6 Profit in North turns to plus



2.7 Beginning of resources increase in South



2.8 Profit in South turns to plus

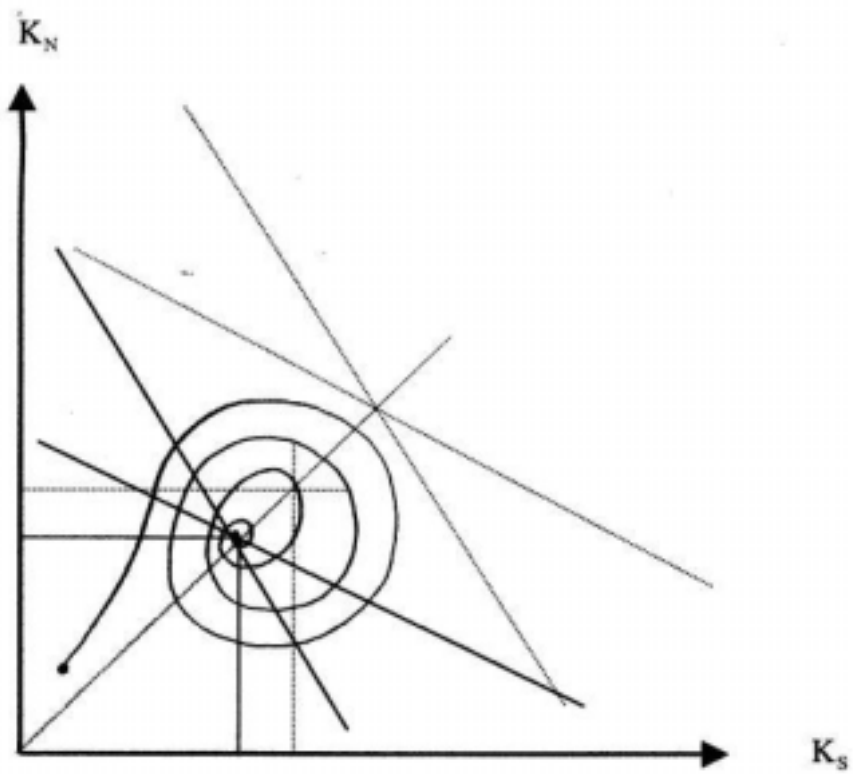
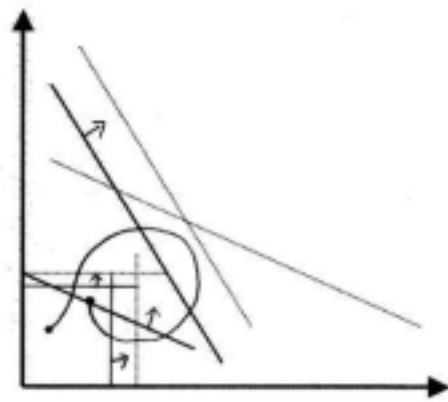


FIGURE 2. Dynamic process to sustainable development

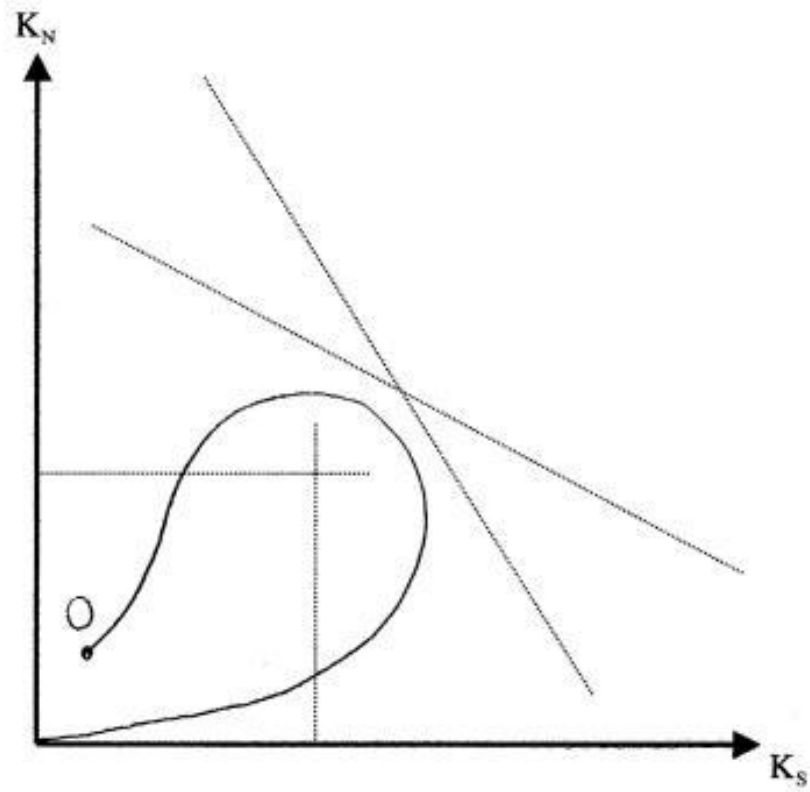


FIGURE 3. Dynamic process in the case of inelastic

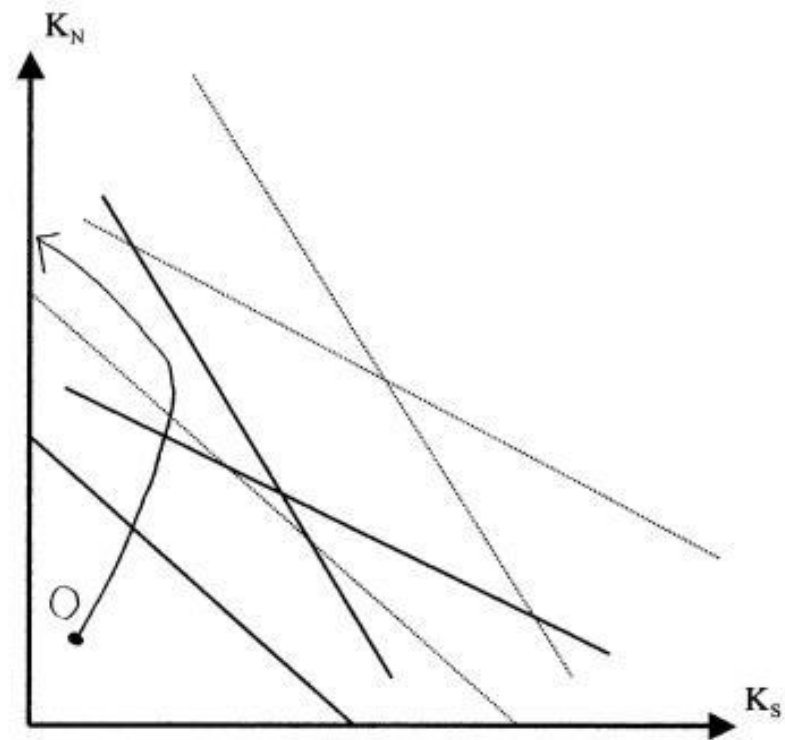


FIGURE 4. Tradable case of renewable resources